Partial order theory: a new setting for the evaluation of multidimensional poverty and inequality

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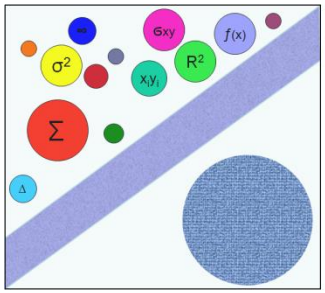


Aim

- To illustrate why the theory of partially ordered sets should be of interest for social scientists, particularly for those involved in multidimensional poverty studies.
- The speech will be kept at a conceptual level, in order to motivate the interest in partial orders.
- Technicalities will be kept at a minimum.

Multidimensional deprivation: complexity and paradox

- Multidimensional deprivation is a **complex** phenomenon, i.e. being deprived involves different aspects and their interactions.
- Multidimensional deprivation may be «**paradoxical**», i.e. positive and negative achievements may be present at the same time in individual experience (e.g. you can have a luxury house downtown and, as a consequence, you may suffer from pollution problems).
- Multidimensional deprivation is naturally described in **qualitative terms** (i.e. often through adverbs: enough, hardly, quite…).

These features reflect into data, which are:

- **Multidimensional** (trivial!).
- Often **not so strongly interdependent, not allowing for one-dimensional reduction**.
- **Ordinal**, not allowing for statistical approaches «attribute driven», i.e. implicitly based on the ideas of «vectors», «linear combinations» and «euclidean vector spaces».

Our point of view on multidimensional deprivation measurement

- Focus should be on **achievement profiles** (sequences of achievements over attributes), rather than on attribute «vectors». In fact, only achievement profiles as a whole globally «capture» the «status» of an individual.

- **Ordinal data must be treated as such**, without assuming «true» numerical scales behind them. No linear combinations of ordinal scores (!!!) allowed, i.e. **no attribute aggregation**.
- Multidimensional deprivation involves both a «**vertical perspective**» (**poverty intensity**) and a «**horizontal perspective**» (people may be poor in different ways and under different perspecitves; **vagueness, nuances and ambiguities in poor identification**).
- As a consequence (and more naturally) deprivation assessment must be addressed as a «**comparison to benchmarks**» problem, rather than as a measure against some absolute scale.
- Exogenous information (e.g. attribute importance) must be introduced respecting the ordinal nature of the scores: **no numerical weights allowed**.

In other words

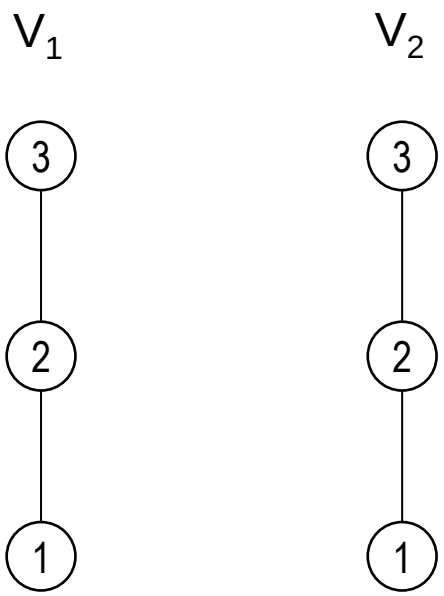
We need a «**purely ordinal**» evaluation process, i.e. a poverty evaluation process which takes the «usual steps» like

1. threshold selection
2. addition of external information
3. production of synthetic indicators
4. …

but involving only «**ordinal concepts**» and only mathematical **tools designed for ordinal data and order relations**.

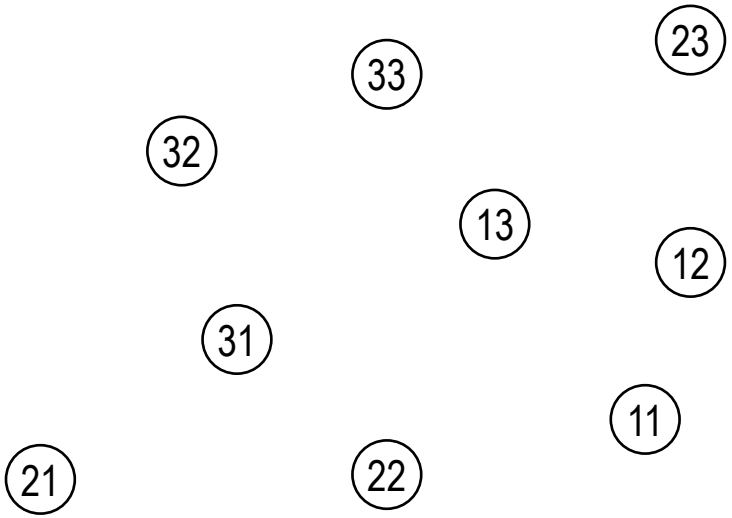
Toy example

Consider 2 variables pertaining to deprivation, with 3 degrees (1, 2, 3).



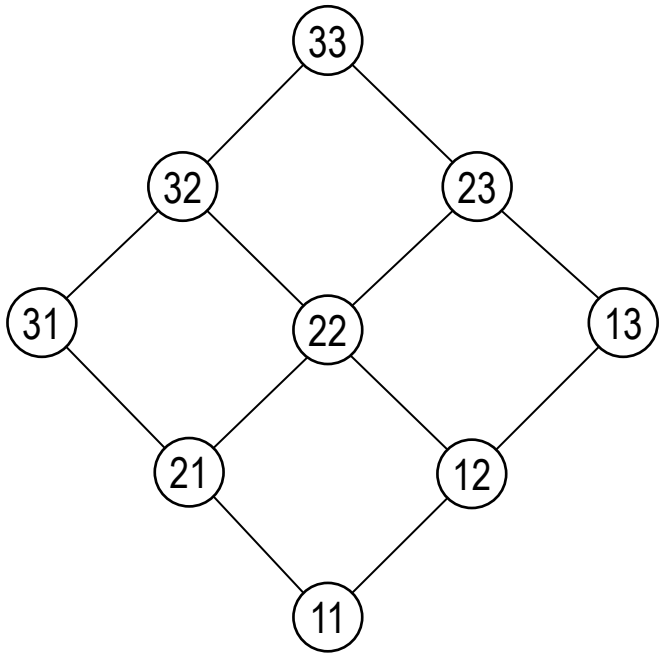
Toy example

There are 9 possible achievement profiles.



Toy example

What can we do with these 9 profiles? Just partially order them, getting the following «space» (a **partially ordered set**, or **poset**, here called the basic achievement poset).



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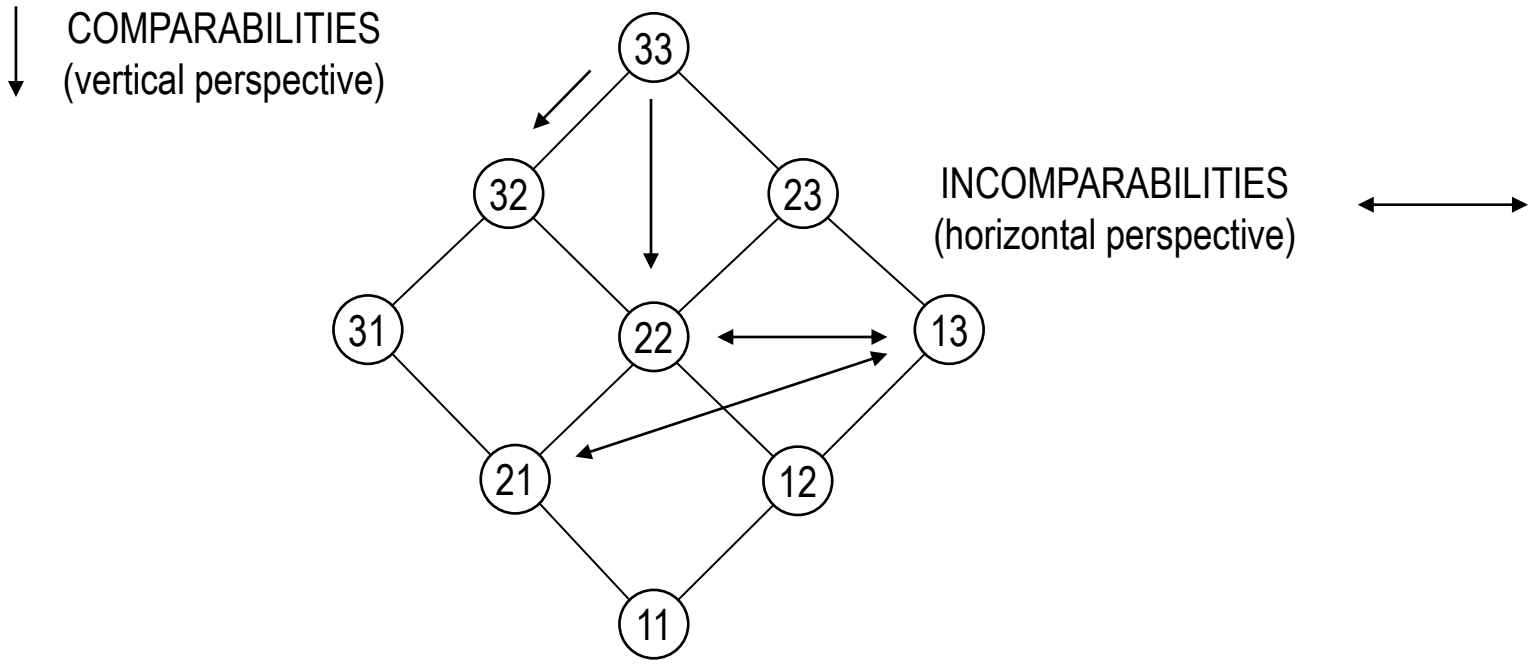


Toy example

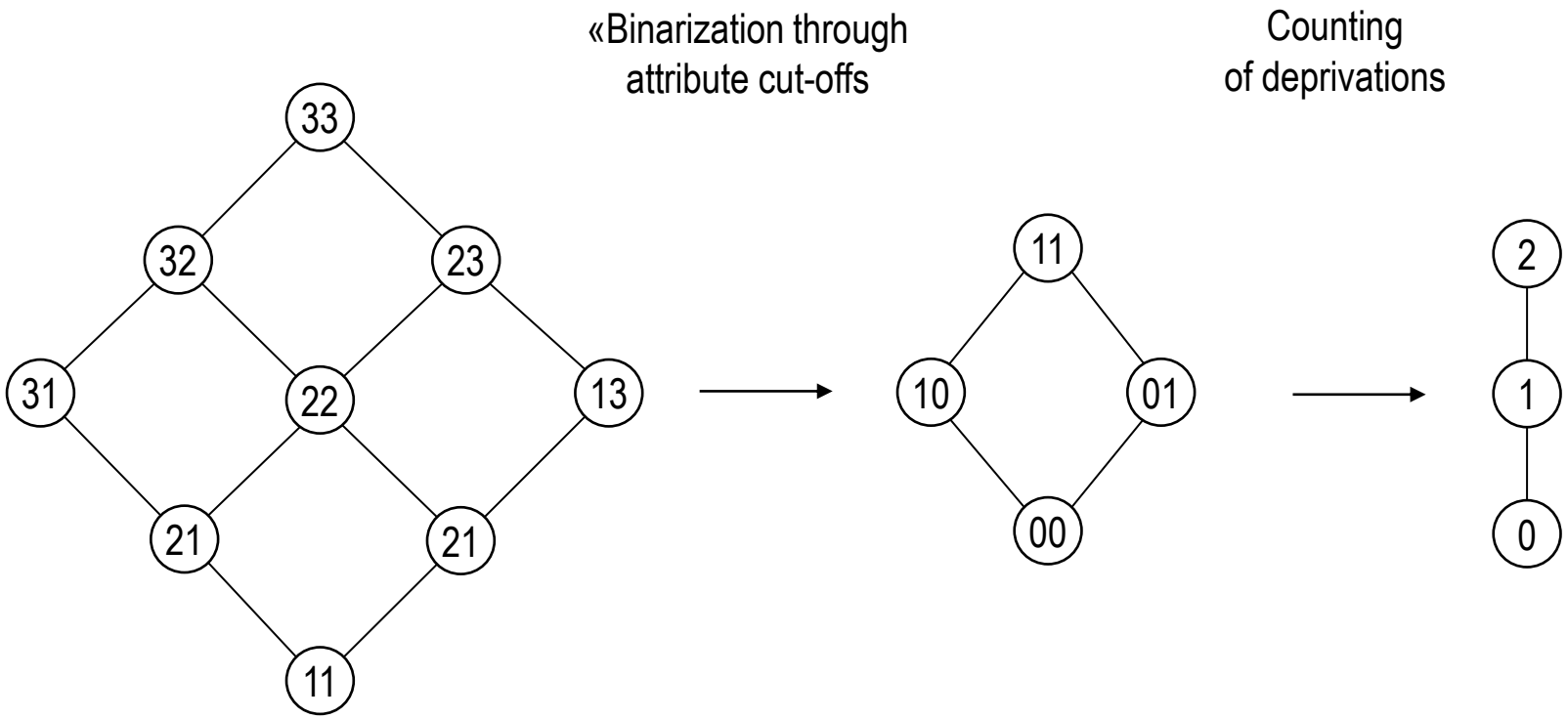
Is it possible to extract information out of such an evaluation space?
If yes, how?

Otherwise stated:

is it possible to build an evaluation process that draws only upon **comparabilities/incomparabilities between achievement profiles**?

Counting Approach

The Counting Approach solves the problem «**collapsing**» incomparabilites, i.e. the partial order structure, into a complete order composed of equivalence classes of profiles.



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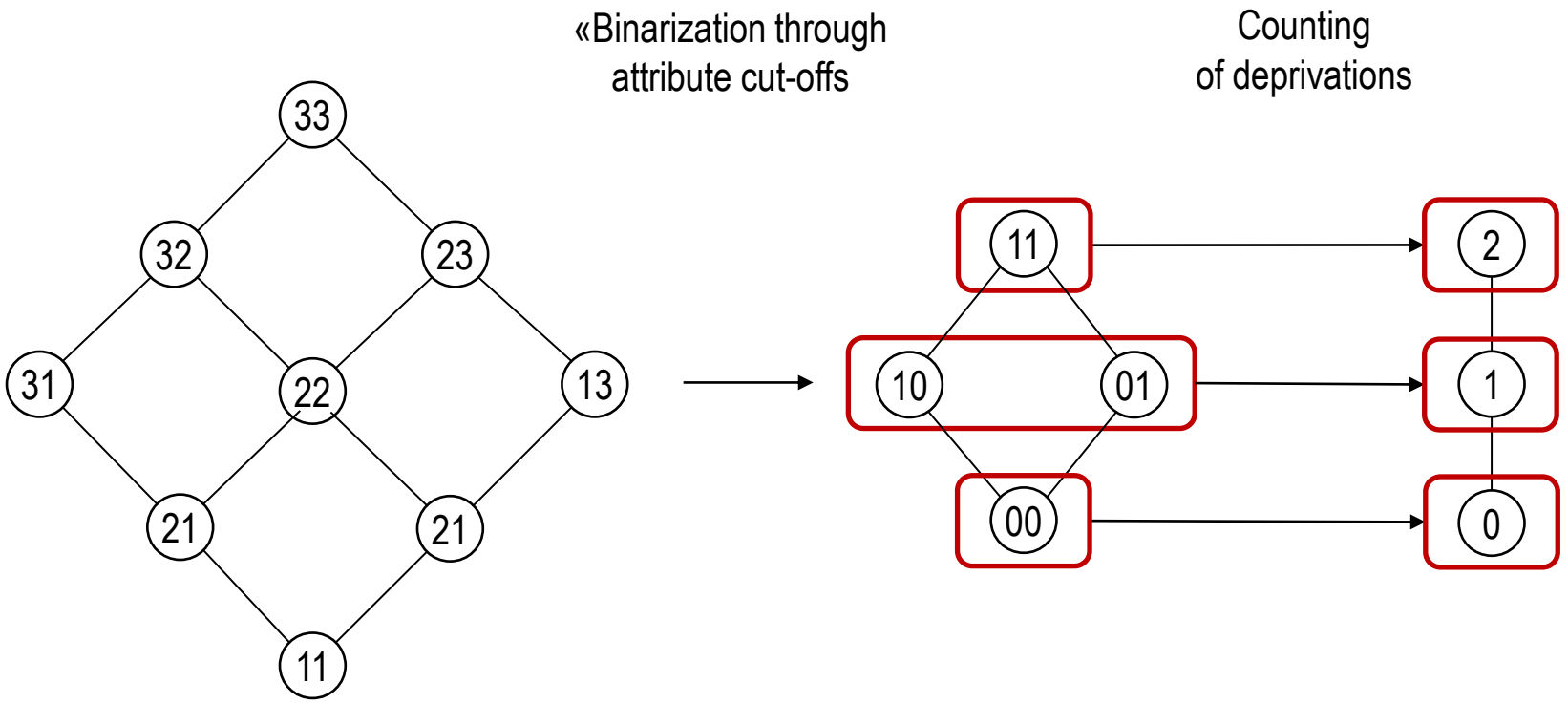


The «power» of partial orders

But:

1. The achievement poset, i.e. the partial order relation defining it, **comprises additional information** on deprivation intensity (comparabilities) and ambiguity or vagueness (incomparabilities).

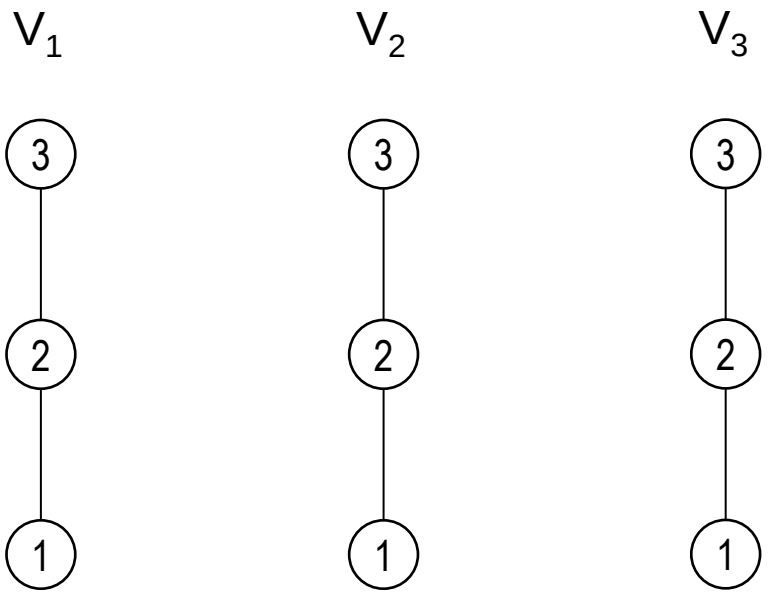
2. Part of this information gets lost in the counting procedure (particularly, but not only, that pertaining to poverty ambiguity and vagueness). In the end, **achievement profiles are classified as «deprived or not deprived», in a binary way**.

3. **Tools exist** to extract this information out of the profile poset.

We now give just some hints.

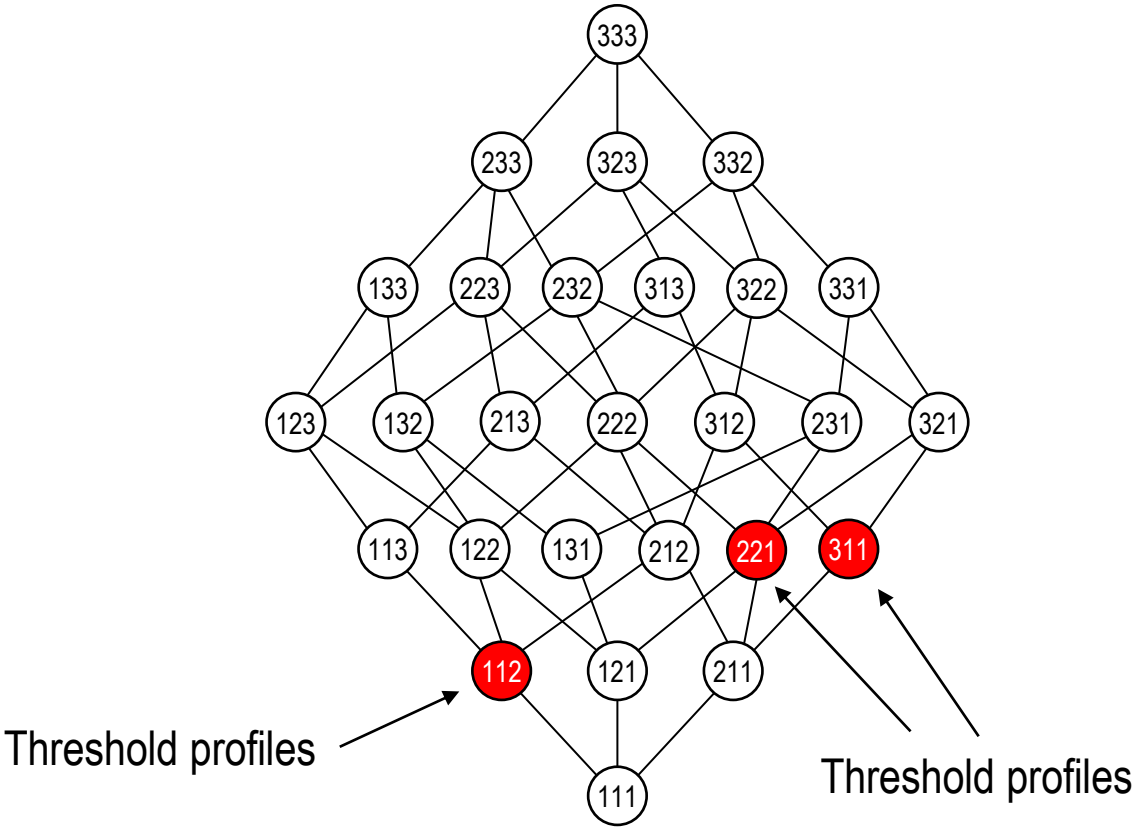
Example

3 attributes on 3 degree scales (1, 2, 3).



Example

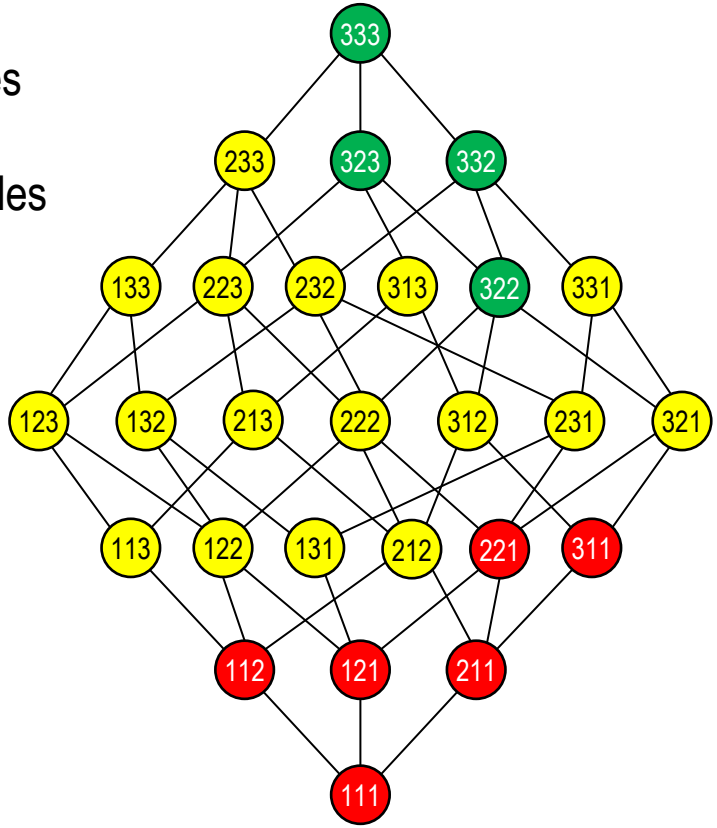
Combining achievements, we get 27 achievement pofiles. and the following achievement poset. On it, a threshold (in terms of profiles, not attributes!) is identified.



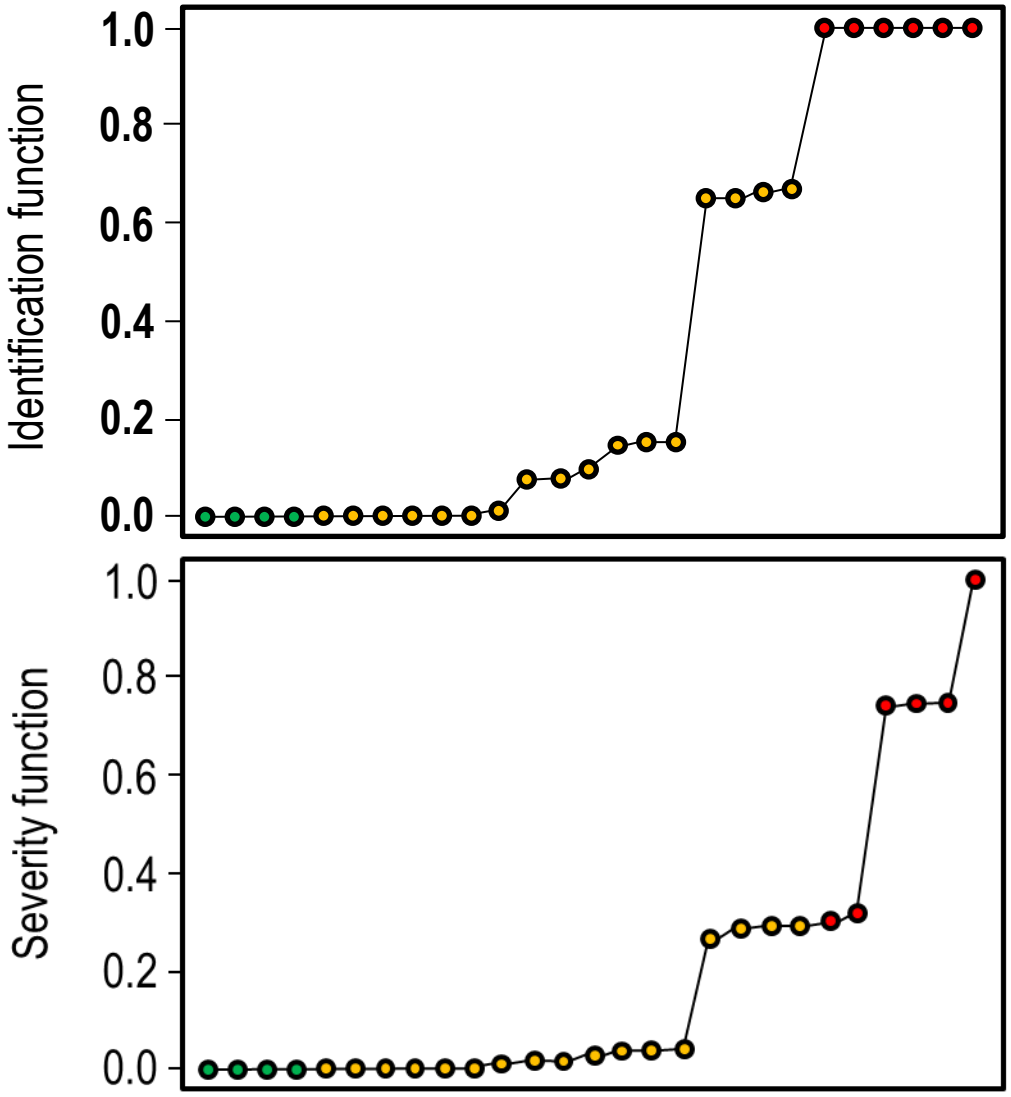
Example

Given the threshold, some profiles are mathematically identified as completely deprived, some others as partly deprived and the remaining as non-deprived.

- Non-deprived profiles
- Partly deprived profiles
- Deprived profiles



Identification and severity functions

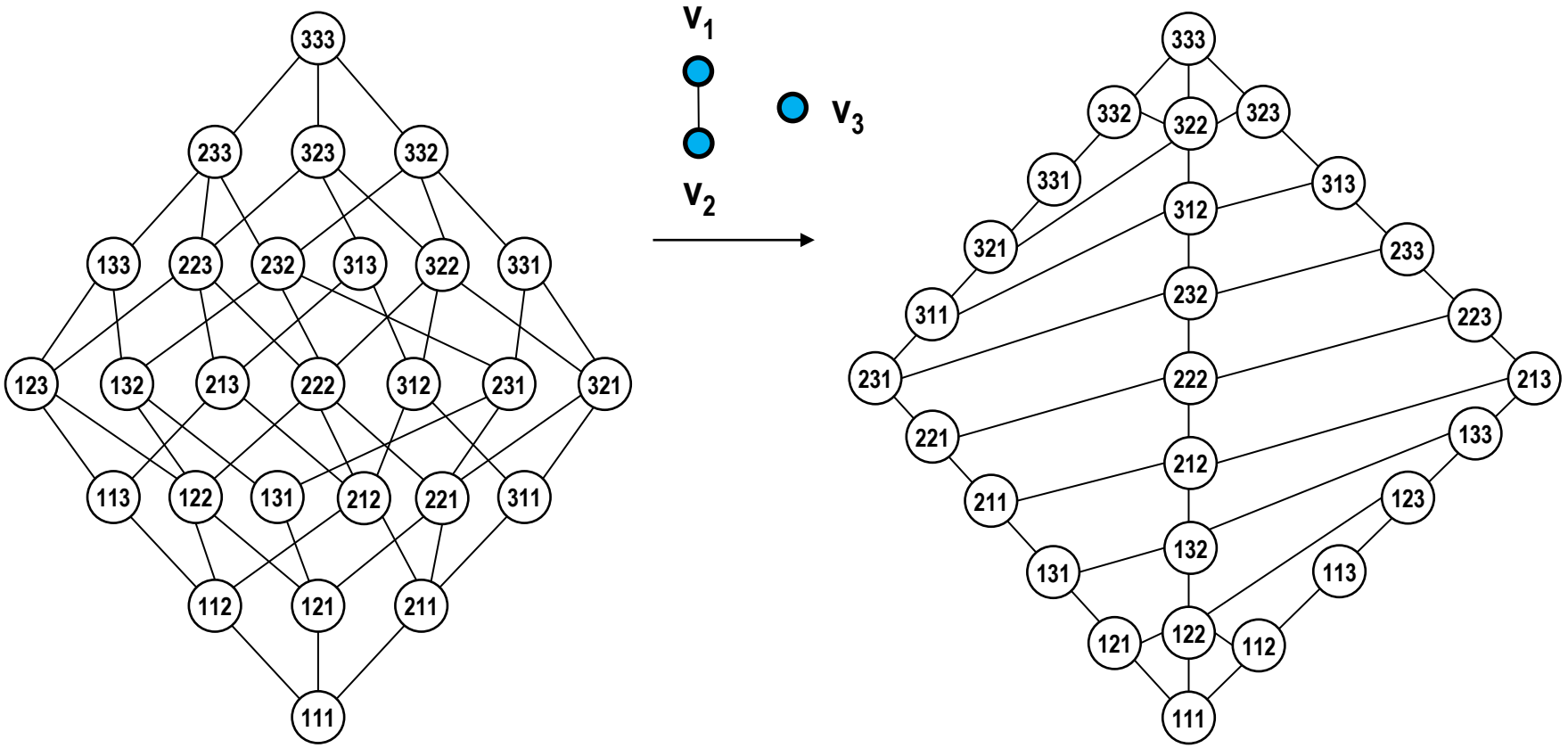


Deprivation scores
(horizontal perspective)
Not binary!

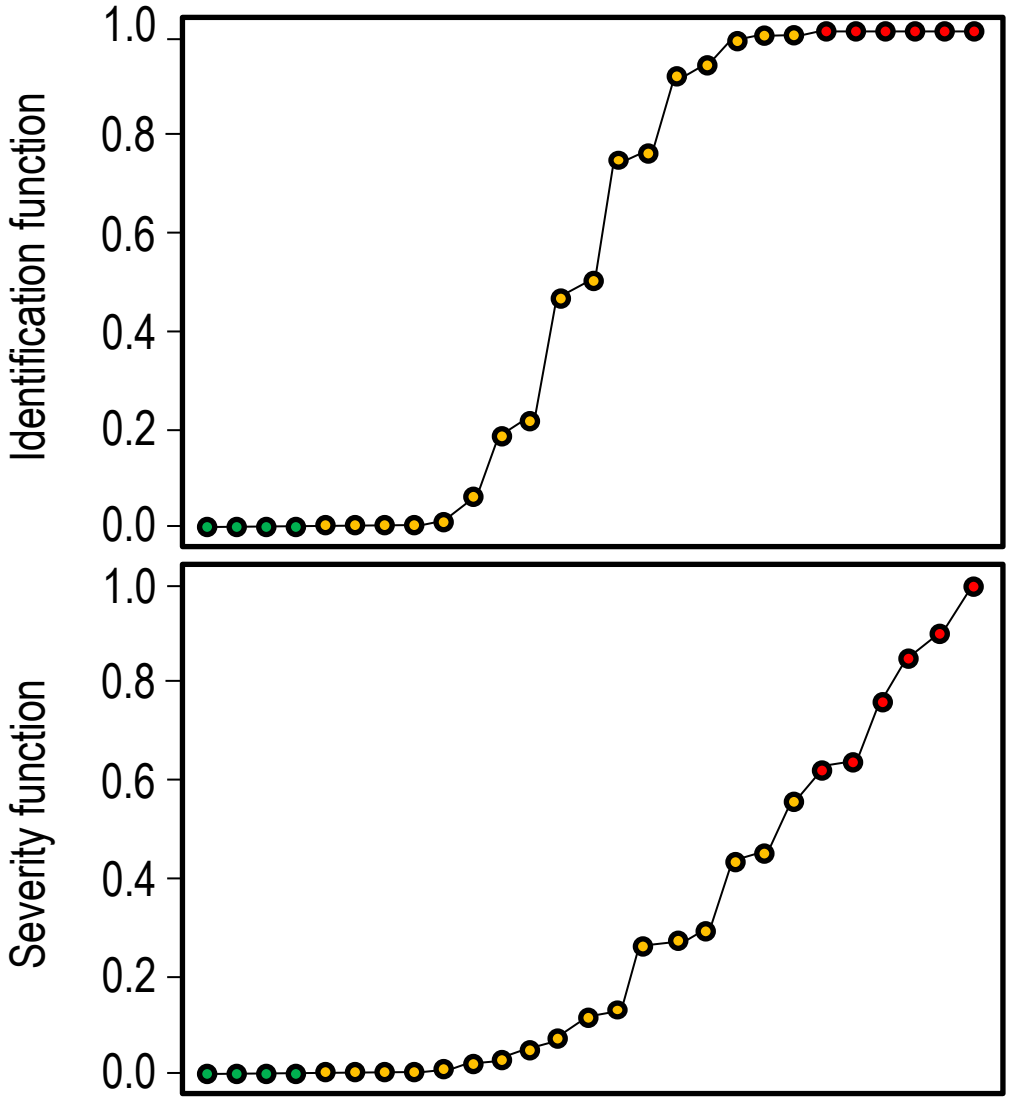
Deprivation severity
(vertical perspective)

Adding external information

How to introduce information on attribute relevance? **changing the structure of the achievement poset** (technically, «extending» it).



(new) Identification and severity functions



Deprivation scores
(horizontal perspective)

Deprivation severity
(vertical perspective)

Comment

The Partial Order Approach:

1. Is the right «conceptual» and mathematical setting to represent deprivation data.

2. Shows that a «purely evaluation procedure» can be implemented.

3. Comprises the Counting Approach as a special case (the Counting Approach is basically the Partial Order Approach applied to a properly extended achievement poset).

4. Is heavy from a computational point of view; this at present limits its applicability (we are working to simplify computations).

5. Should be extended also to numerical attributes, whenever one dimensional reductions are not possible, i.e. whenever data are not strongly interdependent and deprivation evaluation is to be addressed as a multidimensional comparison problem.

Comment

1. Partial order theory is the «grammar of multidimensional ordinal data» and of multidimensional comparison problems, so it should be used in deprivation, well-being and quality-of.life studies.

2. It may also help addressing the measurement of multidimensional ordinal inequality, which should be seen not as the composition of inequalities on single attributes, but as inequality of frequency distributions over achievement posets.

   - We are developing such an approach, using poset and graph theory.

   - The approach seems promising, leading to an effective theory of multidimensional inequality measurement and decomposition.

Thank you

A few references

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